THE SIMPLEST MODEL FOR ILLUSTRATING THE CONIC SECTIONS.

By LEONARD E. DICKSON, B. Sc., Fellow in Pure Mathematics, in the Chicago University.

The Greek geometers prior to Appollonius of Perga supposed that three cones were necessary in forming the conic sections. Thus, the ellipse was cut from an acute-angled cone; the parabola, from a right-angled cone; the hyperbola, from an obtuse-angled cone, each by a plane perpendicular to an edge. The very names, ellipse, parabola, and hyperbola, express the fact that the angle at the vertex of the cone is less than, equal to, or greater than a right angle; and thus that in the ellipse, the cutting plane *falls short* of the other nappe of the cone; in the parabola, is *parallel* to the edge of the cone; in the hyperbolia, *reaches over* to the other nappe of the cone.

Thus the earlier Greeks thought the ellipse peculiar to the acute-angled cone, the parabola to the right-angled cone, the hyperbola to the obtuse-angled cone.

But Appollonius of Perga showed in the year 250 B. C. that all three conics could be cut from single cone by varying the inclination of the cutting plane. Thus he cut a right-circular cone by a plane. Now if the angle between the cutting plane and the base of the cone be *less* than the angle made by an edge of the cone with the base, the section is an *ellipse*; if *equal*, a *parabola*; if *greater* an *hyperbola*.

The method of Appollonius, now used in our models consists in cutting a fixed cone by a revolving plane; my method consists in making the plane fixed and revolving the cone.

Nature furnishes us with a fixed horizontal plane, the surface of a liquid at rest. Filling partly full a hollow glass cone with some liquid (colored to make the effect distinct), we can make at will all the conic sections and their limiting positions. A definite amount of liquid will give a certain series of conics. By varying the quantity of liquid, we can form series of conics magnified in different degrees. Thus we can vary their curvature at will.

Suppose our cone is lying with an edge on a plane. If the plane be horizontal, the section is always a parabola, even if the cone be rolled about on the plane. If the plane is inclined to the horizon at an angle less than the angle at the vertex of the cone, then by rolling the cone about on the plane, the section passes successively through various forms of ellipses, then a parabola, then various forms of hyperbola, and again a parabola, and lastly the original ellipses.

This model has the great superiority over the ordinary wooden models, etc., in that we may readily study by it how the conics pass from one class into another through the limiting forms, the parabola, the circle, and the straight line.

Again, any school boy can make use of his conical ink-bottle, if no better model be at hand.

My method may be hapily extended to the construction of simple models for showing the various sections of any solid whatever.

[Read before the *Texas Academy of Science* in 1892.]